Next-to-Leading Order QCD at the Tevatron and the LHC

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Outline

- Motivation for next-to-leading order
- Overview of the MCFM program
- NLO predictions for vector boson + jets processes
- Vector boson + jets with a single *b*-tag
- Single top production
- Weak boson fusion
- Summary

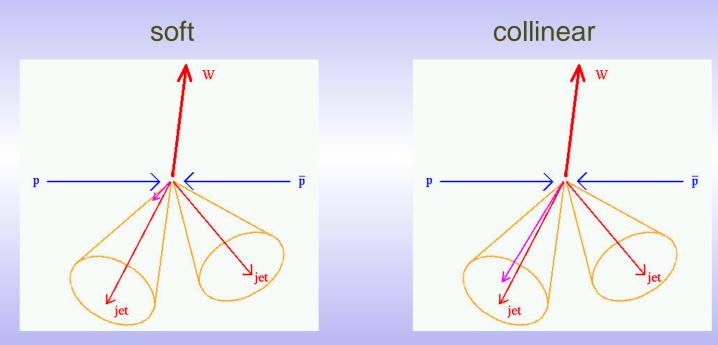
Why NLO?

The benefits of higher order calculations are well known

- Less sensitivity to unphysical input scales
 - ★ first predictive normalization of observables at NLO
 - more accurate estimates of backgrounds for new physics searches and (hopefully) interpretation
 - confidence that cross-sections are under control for precision measurements
- More physics
 - ⋆ jet merging
 - * initial state radiation
 - ⋆ more parton fluxes
- It represents the first step for a plethora of other techniques
 - matching with resummed calculations
 - ⋆ NLO parton showers

Next-to-leading order

At next-to-leading order, we include an extra "unresolved" parton in the final state



■ The theory begins to look more like an experimental jet, so one expects a better agreement with data.

An experimenter's wishlist

Hadron collider cross-sections one would like to know at NLO
Run II Monte Carlo Workshop, April 2001

Single boson	Diboson	Triboson	Heavy flavour
$W+\leq 5j$	$WW + \leq 5j$	$WWW + \leq 3j$	$t\bar{t} + \leq 3j$
$W + b\bar{b} + \leq 3j$	$WW + b\bar{b} + \leq 3j$	$WWW + b\bar{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\overline{c} + \leq 3j$	$WW + c\overline{c} + \leq 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \leq 2j$
$Z + \leq 5j$	$ZZ + \leq 5j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \leq 2j$
$Z + b\bar{b} + \leq 3j$	$ZZ + b\bar{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \leq 2j$
$Z + c\bar{c} + \leq 3j$	$ZZ + c\overline{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 2j$
$\gamma + \leq 5j$	$\gamma\gamma + \leq 5j$		$b\bar{b} + \leq 3j$
$\gamma + b\bar{b} + \leq 3j$	$\gamma\gamma + b\bar{b} + \leq 3j$		
$\gamma + c\bar{c} + \leq 3j$	$\gamma\gamma + c\bar{c} + \leq 3j$		
	$WZ + \leq 5j$		
	$WZ + b\bar{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 3j$		
	$Z\gamma + \leq 3j$		

Theoretical status

Single boson	Diboson	Triboson	Heavy flavour
$W + \leq 2j$	$WW + \leq 0j$	$WWW + \leq 3j$	$t\bar{t} + \leq 0j$
$W + b\overline{b} + \leq 0j$	$WW + b\overline{b} + \le 3j$	$WWW + b\overline{b} + \leq 3j$	$t\bar{t} + \gamma + \leq 2j$
$W + c\bar{c} + \le 0j$	$WW + c\overline{c} + \le 3j$	$WWW + \gamma\gamma + \leq 3j$	$t\bar{t} + W + \le 2j$
$Z + \leq 2j$	$ZZ + \leq 0j$	$Z\gamma\gamma + \leq 3j$	$t\bar{t} + Z + \le 2j$
$Z + b\bar{b} + \le 0j$	$ZZ + b\overline{b} + \leq 3j$	$WZZ + \leq 3j$	$t\bar{t} + H + \le 0j$
$Z + c\bar{c} + \le 0j$	$ZZ + c\overline{c} + \leq 3j$	$ZZZ + \leq 3j$	$t\bar{b} + \leq 0j$
$\gamma + \leq 1j$	$\gamma\gamma + \leq 1j$		$b\bar{b} + \leq 0j$
$ \begin{array}{c} \gamma + b\overline{b} + \leq 3j \\ \gamma + c\overline{c} + \leq 3j \end{array} $	$\begin{array}{c} \gamma\gamma + b\overline{b} + \leq 3j \\ \gamma\gamma + c\overline{c} + \leq 3j \end{array}$		
	$WZ + \leq 0j$		
	$WZ + b\overline{b} + \leq 3j$		
	$WZ + c\bar{c} + \leq 3j$		
	$W\gamma + \leq 0j$		
	$Z\gamma + \leq 0j$		



MCFM philosophy

- Put together a collection of NLO calculations in a general purpose code
- Take advantage of existing matrix element results in the literature where possible; several new calculations have also been implemented
- There are generic routines for handling common tasks such as the subtraction of singularities, so that implementing new processes is "painless"
- Emphasis has been on bringing together calculations of signals and backgrounds for particularly challenging searches, so that NLO effects may be more easily studied with just one code
- Where possible, appropriate decays of vector bosons are included and all possible spin correlations are retained for a better assessment of the effect of experimental cuts

MCFM overview

JC and R.K. Ellis

 \blacksquare Parton level cross-sections predicted to NLO in α_S

$$\begin{array}{|c|c|c|}\hline p\bar{p} \rightarrow W^{\pm}/Z & p\bar{p} \rightarrow W^{+} + W^{-} \\ p\bar{p} \rightarrow W^{\pm} + Z & p\bar{p} \rightarrow Z + Z \\ p\bar{p} \rightarrow W^{\pm} + \gamma & p\bar{p} \rightarrow W^{\pm}/Z + H \\ p\bar{p} \rightarrow W^{\pm} + g^{\star} (\rightarrow b\bar{b}) & p\bar{p} \rightarrow Zb\bar{b} \\ p\bar{p} \rightarrow W^{\pm}/Z + 1 \text{ jet} & p\bar{p} \rightarrow W^{\pm}/Z + 2 \text{ jets} \\ p\bar{p}(gg) \rightarrow H & p\bar{p}(gg) \rightarrow H + 1 \text{ jet} \\ p\bar{p} \rightarrow H + b & p\bar{p} \rightarrow Z + b \end{array}$$

low particle multiplicity (no showering)

(+F. Tramontano)

(+F. Maltoni, S. Willenbrock)

- no hadronization
- hard to model detector effects
- \oplus less sensitivity to μ_R , μ_F
- rates are better normalized
- fully differential distributions

process.DAT fragment

$ \begin{array}{ c c c c }\hline 1 & W^+(\to\nu(p_3)+e^+(p_4)) & NLO \\ 6 & W^-(\to e^-(p_3)+\bar{v}(p_4)) & NLO \\ \hline 111 & W^+(\to\nu(p_3)+e^+(p_4))+f(p_5) & NLO \\ 12 & W^+(\to\nu(p_3)+e^+(p_4))+\bar{c}(p_5) & NLO \\ 13 & W^+(\to\nu(p_3)+e^+(p_4))+\bar{c}(p_5) & NLO \\ 14 & W^+(\to\nu(p_3)+e^+(p_4))+\bar{c}(p_5) & NLO \\ 16 & W^-(\to e^-(p_3)+\bar{v}(p_4))+f(p_5) & NLO \\ 17 & W^-(\to e^-(p_3)+\bar{v}(p_4))+c(p_5) & NLO \\ 18 & W^-(\to e^-(p_3)+\bar{v}(p_4))+c(p_5) & NLO \\ 19 & W^-(\to e^-(p_3)+\bar{v}(p_4))+c(p_5) & NLO \\ 20 & W^+(\to\nu(p_3)+e^+(p_4))+b(p_5)+\bar{b}(p_6) & NLO \\ 21 & W^+(\to\nu(p_3)+e^+(p_4))+b(p_5)+\bar{b}(p_6) & NLO \\ 22 & W^+(\to\nu(p_3)+e^+(p_4))+b(p_5)+\bar{b}(p_6) & NLO \\ 23 & W^+(\to\nu(p_3)+e^+(p_4))+b(p_5)+\bar{b}(p_6)+f(p_7) & NLO \\ 24 & W^+(\to\nu(p_3)+e^+(p_4))+b(p_5)+\bar{b}(p_6)+f(p_7) & LO \\ 25 & W^-(\to e^-(p_3)+\bar{\nu}(p_4))+b(p_5)+\bar{b}(p_6) & NLO \\ 26 & W^-(\to e^-(p_3)+\bar{\nu}(p_4))+b(p_5)+\bar{b}(p_6) & NLO \\ 27 & W^-(\to e^-(p_3)+\bar{\nu}(p_4))+b(p_5)+\bar{b}(p_6) & NLO \\ 28 & W^-(\to e^-(p_3)+\bar{\nu}(p_4))+f(p_5)+f(p_6) & NLO \\ 29 & W^-(\to e^-(p_3)+\bar{\nu}(p_4))+f(p_5)+f(p_6) & NLO \\ 29 & W^-(\to e^-(p_3)+\bar{\nu}(p_4))+f(p_5)+\bar{b}(p_6) & NLO \\ 31 & Z^0(\to e^-(p_3)+\bar{\nu}(p_4))+f(p_5)+\bar{b}(p_6)+f(p_7) & LO \\ 29 & W^-(\to e^-(p_3)+\bar{\nu}(p_4))+f(p_5)+\bar{b}(p_6)+f(p_7) & NLO \\ 41 & Z^0(\to e^-(p_3)+\bar{\nu}(p_4))+f(p_5)+\bar{b}(p_6)+f(p_7) & NLO \\ 42 & Z_0(\to 3\times(\nu(p_3)+\bar{\nu}(p_4))) & NLO \\ 43 & Z^0(\to 3\times(\nu(p_3)+\bar{\nu}(p_4)))-f(p_5)+f(p_6)+f(p_7) & NLO \\ 44 & Z^0(\to e^-(p_3)+e^+(p_4))+f(p_5)+f(p_6)+f(p_7) & NLO \\ 45 & Z^0(\to e^-(p_3)+e^+(p_4))+f(p_5)+f(p_6)+f(p_7) & NLO \\ 46 & Z^0(\to e^-(p_3)+e^+(p_4))+f(p_5)+f(p_6)+f(p_7) & NLO \\ 47 & Z^0(\to e^-(p_3)+e^+(p_4))+f(p_5)+f(p_6)+f(p_7) & NLO \\ 48 & Z^0(\to e^-(p_3)+e^+(p_4))+f(p_5)+f(p_6)+f(p_7) & NLO \\ 49 & Z^0(\to e^-(p_3)+e^+(p_4))+f(p_5)+f(p_6)+f(p_7) & NLO \\ 50 & Z^0(\to e^-(p_3)+e^+(p_4))+f(p_5)+f(p_6)+f(p_7) & NLO \\ 50 & Z^0(\to e^-(p_3)+e^+(p_4))+f(p_5)+\bar{b}(p_6) & NLO \\ 50 & Z^0(\to $	nproc	$f(p_1) + f(p_2) \rightarrow \dots$	Order
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$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	17	$W^{-}(\rightarrow e^{-}(p_3) + \bar{\nu}(p_4)) + \gamma(p_5)$	NLO
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53 $Z^0(\rightarrow b(p_3) + \bar{b}(p_4)) + b(p_5) + \bar{b}(p_6)$ NLO			_
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56 $Z^{0}(\rightarrow e^{-}(p_{3}) + e^{+}(p_{4})) + b(p_{5}) + b(p_{6}) + f(p_{7})$ LO			_
	56	$Z^{0}(\rightarrow e^{-}(p_{3}) + e^{+}(p_{4})) + b(p_{5}) + b(p_{6}) + f(p_{7})$	LO

61 W 62 W 63 W 64 W 71 W 72 W 73 W 76 W 77 W 78 W 81 Z 83 Z 84 Z 84 Z 88 Z 88 Z 89 Z 91 W 96 W	$\begin{array}{l} (p_1) + f(p_2) \to \dots \\ \gamma^+ (\to \nu(p_3) + e^+(p_4)) + W^-(\to e^-(p_5) + \bar{\nu}(p_6)) \\ \gamma^+ (\to \nu(p_3) + e^+(p_4)) + W^-(\to q(p_5) + \bar{q}(p_6)) \\ \gamma^+ (\to q(p_3) + \bar{q}(p_4)) + W^-(\to e^-(p_5) + \bar{\nu}(p_6)) \\ \gamma^+ (\to \nu(p_3) + e^+(p_4)) + W^-(\to e^-(p_5) + \bar{\nu}(p_6)) \\ \gamma^+ (\to \nu(p_3) + \mu^+(p_4)) + Z^0(\to e^-(p_5) + \bar{\nu}(p_6)) \\ \gamma^+ (\to \nu(p_3) + \mu^+(p_4)) + Z^0(\to \nu_e(p_5) + \bar{\nu}_e(p_6)) \\ \gamma^+ (\to \nu(p_3) + \mu^+(p_4)) + Z^0(\to b(p_5) + \bar{b}(p_6)) \\ \gamma^- (\to \mu^-(p_3) + \bar{\nu}(p_4)) + Z^0(\to b(p_5) + \bar{b}(p_6)) \\ \gamma^- (\to e^-(p_3) + \bar{\nu}(p_4)) + Z^0(\to e^-(p_5) + e^+(p_6)) \\ \gamma^- (\to e^-(p_3) + \bar{\nu}(p_4)) + Z^0(\to b(p_5) + \bar{b}(p_6)) \\ \gamma^- (\to e^-(p_3) + \bar{\nu}(p_4)) + Z^0(\to e^-(p_5) + e^+(p_6)) \\ \gamma^- (\to e^-(p_3) + \bar{\nu}(p_4)) + Z^0(\to e^-(p_5) + e^+(p_6)) \\ \gamma^- (\to e^-(p_5) + e^+(p_6)) + Z^0(\to b(p_3) + \bar{b}(p_4)) \\ \gamma^- (\to e^-(p_5) + e^+(p_6)) + Z^0(\to b(p_3) + \bar{b}(p_4)) \\ \gamma^- (\to e^-(p_5) + e^+(p_6)) + Z^0(\to \mu^-(p_3) + \mu^+(p_4)) [\text{In } \gamma^*] \\ \gamma^- (\to e^-(p_5) + e^+(p_6)) + Z^0(\to a^-(p_3) + \mu^+(p_4)) [\text{In } \gamma^*] \\ \gamma^- (\to e^-(p_5) + e^+(p_6)) + Z^0(\to a^-(p_3) + \mu^+(p_4)) [\text{In } \gamma^*] \\ \gamma^- (\to e^-(p_5) + e^+(p_6)) + Z^0(\to a^-(p_3) + \mu^+(p_4)) [\text{In } \gamma^*] \\ \gamma^- (\to e^-(p_5) + e^+(p_6)) + Z^0(\to a^-(p_3) + \mu^+(p_4)) [\text{In } \gamma^*] \\ \gamma^- (\to e^-(p_5) + e^+(p_6)) + Z^0(\to a^-(p_3) + \mu^+(p_4)) [\text{In } \gamma^*] \\ \gamma^- (\to e^-(p_5) + e^+(p_6)) + Z^0(\to a^-(p_3) + \mu^+(p_4)) [\text{In } \gamma^*] \\ \gamma^- (\to e^-(p_5) + e^+(p_6)) + Z^0(\to a^-(p_5) + e^-(p_5) + e^-(p_5)) [\text{In } \gamma^*] \\ \gamma^- (\to e^-(p_5) + e^+(p_6)) + Z^0(\to a^-(p_5) + e^-(p_5) + e^-(p_5)) [\text{In } \gamma^*] \\ \gamma^- (\to e^-(p_5) + e^-(p$	NLO
62 W 63 W 71 W 72 W 73 W 76 W 77 W 81 Z 82 Z 83 Z 86 Z 87 Z 88 Z 88 Z 89 Z 91 W 96 W	$\begin{array}{l} V^{+}(\rightarrow\nu(p_{3})+e^{+}(p_{4}))+W^{-}(\rightarrow q(p_{5})+\bar{q}(p_{6}))\\ V^{+}(\rightarrow q(p_{3})+\bar{q}(p_{4}))+W^{-}(\rightarrow e^{-}(p_{5})+\bar{\nu}(p_{6}))\\ V^{+}(\rightarrow\nu(p_{3})+e^{+}(p_{4}))+W^{-}(\rightarrow e^{-}(p_{5})+\bar{\nu}(p_{6})) \text{ [no pol]}\\ V^{+}(\rightarrow\nu(p_{3})+\mu^{+}(p_{4}))+Z^{0}(\rightarrow e^{-}(p_{5})+\bar{\nu}(p_{6}))\\ V^{+}(\rightarrow\nu(p_{3})+\mu^{+}(p_{4}))+Z^{0}(\rightarrow e^{-}(p_{5})+\bar{\nu}(p_{6}))\\ V^{+}(\rightarrow\nu(p_{3})+\mu^{+}(p_{4}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ V^{-}(\rightarrow\mu^{-}(p_{3})+\bar{\nu}(p_{4}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ V^{-}(\rightarrow e^{-}(p_{3})+\bar{\nu}(p_{4}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ V^{-}(\rightarrow e^{-}(p_{3})+\bar{\nu}(p_{4}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ V^{-}(\rightarrow e^{-}(p_{3})+\bar{\nu}(p_{4}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ V^{-}(\rightarrow e^{-}(p_{3})+\bar{\nu}(p_{4}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ V^{-}(\rightarrow e^{-}(p_{5})+\bar{\nu}(p_{4}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{3})+\bar{\nu}(p_{4}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{4}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))\\ V^{-}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6})+\mu^{+}(p_{6})+\mu^{+}(p_{6})\\ V^{-}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6})+\mu^{+}(p_{6})+\mu^{+}(p_{6})\\ $	NLO
63	$\begin{array}{l} V^{+}(\to q(p_3) + \bar{q}(p_4))' + W^{-}(\to e^{-}(p_5) + \bar{\nu}(p_6)) \\ V^{+}(\to \nu(p_3) + e^{+}(p_4)) + W^{-}(\to e^{-}(p_5) + \bar{\nu}(p_6)) \text{ [no pol]} \\ V^{+}(\to \nu(p_3) + e^{+}(p_4)) + W^{-}(\to e^{-}(p_5) + e^{+}(p_6)) \\ V^{+}(\to \nu(p_3) + \mu^{+}(p_4)) + Z^{0}(\to e^{-}(p_5) + e^{+}(p_6)) \\ V^{+}(\to \nu(p_3) + \mu^{+}(p_4)) + Z^{0}(\to \nu(p_5) + \bar{\nu}(p_6)) \\ V^{-}(\to \mu^{-}(p_3) + \bar{\nu}(p_4)) + Z^{0}(\to e^{-}(p_5) + e^{+}(p_6)) \\ V^{-}(\to e^{-}(p_3) + \bar{\nu}(p_4)) + Z^{0}(\to \nu(p_5) + \bar{\nu}(p_6)) \\ V^{-}(\to e^{-}(p_3) + \bar{\nu}(p_4)) + Z^{0}(\to b(p_5) + \bar{\nu}(p_6)) \\ V^{-}(\to e^{-}(p_3) + \bar{\nu}(p_4)) + Z^{0}(\to e^{-}(p_5) + e^{+}(p_6)) \\ V^{-}(\to e^{-}(p_3) + \bar{\nu}(p_4)) + Z^{0}(\to e^{-}(p_5) + e^{+}(p_6)) \\ V^{-}(\to e^{-}(p_5) + e^{+}(p_6)) + Z^{0}(\to b(p_3) + \bar{b}(p_4)) \\ V^{-}(\to e^{-}(p_5) + e^{+}(p_6)) + Z^{0}(\to b(p_3) + \bar{b}(p_4)) \\ V^{-}(\to e^{-}(p_5) + e^{+}(p_6)) + Z^{0}(\to b(p_3) + \bar{b}(p_6)) \\ V^{-}(\to e^{-}(p_5) + e^{+}(p_6)) + Z^{0}(\to b(p_3) + \bar{\mu}(p_4)) \\ V^{-}(\to e^{-}(p_5) + e^{+}(p_6)) + Z^{0}(\to b(p_3) + \mu^{+}(p_4)) \\ V^{-}(\to e^{-}(p_5) + e^{+}(p_6)) + Z^{0}(\to b(p_3) + \mu^{+}(p_4)) \\ V^{-}(\to e^{-}(p_5) + e^{+}(p_6)) + Z^{0}(\to b(p_5) + \mu^{+}(p_4)) \\ V^{-}(\to e^{-}(p_5) + e^{+}(p_6)) + Z^{0}(\to b(p_5) + \mu^{+}(p_4)) \\ V^{-}(\to e^{-}(p_5) + e^{+}(p_6)) + Z^{0}(\to b(p_5) + \mu^{+}(p_4)) \\ V^{-}(\to e^{-}(p_5) + e^{+}(p_6)) + Z^{0}(\to b(p_5) + \mu^{+}(p_4)) \\ V^{-}(\to e^{-}(p_5) + e^{+}(p_6)) + Z^{0}(\to b^{-}(p_5) + \mu^{+}(p_4)) \\ V^{-}(\to e^{-}(p_5) + e^{+}(p_6)) + Z^{0}(\to b^{-}(p_5) + \mu^{+}(p_4)) \\ V^{-}(\to e^{-}(p_5) + e^{+}(p_6)) + Z^{0}(\to b^{-}(p_5) + \mu^{+}(p_4)) \\ V^{-}(\to e^{-}(p_5) + e^{+}(p_6)) + Z^{0}(\to b^{-}(p_5) + \mu^{+}(p_4)) \\ V^{-}(\to e^{-}(p_5) + \mu^{+}(p_6)) + Z^{0}(\to e^{-}(p_5) + \mu^{+}(p_6) \\ V^{-}(\to e^{-}(p_5) + \mu^{+}(p_6)) + Z^{0}(\to e^{-}(p_5) + \mu^{+}(p_6) \\ V^{-}(\to e^{-}(p_5) + \mu^{+}(p_6)) + Z^{0}(\to e^{-}(p_5) + \mu^{+}(p_6) \\ V^{-}(\to e^{-}(p_5) + \mu^{+}(p_6)) + Z^{0}(\to e^{-}(p_5) + \mu^{+}(p_6) \\ V^{-}(\to e^{-}(p_5) + \mu^{+}(p_6) \\ V^{-}(\to e^{-}(p_5) + \mu^{+}(p_6) + \mu^{+}(p_6) \\ V^{-}(\to e^{-}(p_5) + \mu^{+}(p_6) + \mu^{+}(p_6) \\ V^{-}(\to e^{-}(p_5) + \mu^{+}(p$	NLO NLO NLO NLO NLO NLO NLO NLO NLO NLO
64 W 71 W 72 W 73 W 76 W 77 W 78 W 81 Z 82 Z 83 Z 84 Z 86 Z 87 Z 88 Z 91 W 96 W	$\begin{array}{l} v^{+}(\rightarrow\nu(p_{3})+e^{+}(p_{1}))+W^{-}(\rightarrow e^{-}(p_{5})+\bar{\nu}(p_{6})) & \text{[no pol]} \\ v^{+}(\rightarrow\nu(p_{3})+\mu^{+}(p_{6}))+Z^{0}(\rightarrow e^{-}(p_{5})+e^{+}(p_{6})) \\ v^{+}(\rightarrow\nu(p_{3})+\mu^{+}(p_{4}))+Z^{0}(\rightarrow\nu_{e}(p_{5})+\bar{\nu}_{e}(p_{5})) \\ v^{+}(\rightarrow\nu(p_{3})+\mu^{+}(p_{4}))+Z^{0}(\rightarrow\nu_{e}(p_{5})+\bar{\nu}_{e}(p_{5})) \\ v^{-}(\rightarrow\mu^{-}(p_{3})+\bar{\nu}(p_{4}))+Z^{0}(\rightarrow e^{-}(p_{5})+e^{+}(p_{6})) \\ v^{-}(\rightarrow e^{-}(p_{3})+\bar{\nu}(p_{4}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6})) \\ v^{-}(\rightarrow e^{-}(p_{3})+\bar{\nu}(p_{4}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6})) \\ v^{-}(\rightarrow e^{-}(p_{3})+\mu^{+}(p_{4}))+Z^{0}(\rightarrow e^{-}(p_{5})+e^{+}(p_{6})) \\ v^{0}(\rightarrow 3\times(\nu(p_{3})+\bar{\nu}(p_{4})))+Z^{0}(\rightarrow e^{-}(p_{5})+e^{+}(p_{6})) \\ v^{0}(\rightarrow e^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{3})+\bar{\nu}(p_{4})) \\ v^{0}(\rightarrow s\times(\nu(p_{3})+\bar{\nu}(p_{4})))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6})) \\ v^{0}(\rightarrow e^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{3})+\mu^{+}(p_{4})) \\ v^{0}(\rightarrow e^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{3})+\mu^{+}(p_{4})) \\ v^{0}(\rightarrow e^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{3})+\mu^{+}(p_{4})) \\ v^{0}(\rightarrow\nu(p_{3})+\bar{\nu}(p_{4}))+Z^{0}(\rightarrow\nu(p_{3})+\mu^{+}(p_{4})) \\ v^{0}(\rightarrow\nu(p_{3})+\bar{\nu}(p_{4}))+Z^{0}(\rightarrow\nu(p_{3})+\bar{\nu}(p_{4})) \\ v^{0}(\rightarrow\nu(p_{3})+\bar{\nu}(p_{4}))+Z^{0}(\rightarrow\nu(p_{3})+\bar{\nu}($	NLO
71 W 72 W 73 W 76 W 77 W 81 Z 82 Z 83 Z 84 Z 86 Z 87 Z 88 Z 89 Z 91 W 96 W	$\begin{array}{l} \lambda^{+}(\rightarrow\nu(p_{3})+\mu^{+}(p_{k}))+Z^{0}(\rightarrow e^{-}(p_{5})+e^{+}(p_{6}))\\ \lambda^{+}(\rightarrow\nu(p_{3})+\mu^{+}(p_{k}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}_{e}(p_{6}))\\ \lambda^{+}(\rightarrow\nu(p_{3})+\mu^{+}(p_{k}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}_{e}(p_{6}))\\ \lambda^{+}(\rightarrow\nu(p_{3})+\mu^{+}(p_{k}))+Z^{0}(\rightarrow e^{-}(p_{5})+e^{+}(p_{6}))\\ \lambda^{-}(\rightarrow\mu^{-}(p_{3})+\bar{\nu}(p_{k}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ \lambda^{-}(\rightarrow e^{-}(p_{3})+\bar{\nu}(p_{k}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ \lambda^{-}(\rightarrow e^{-}(p_{3})+\bar{\nu}(p_{k}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ \lambda^{-}(\rightarrow\mu^{-}(p_{3})+\mu^{+}(p_{k}))+Z^{0}(\rightarrow\nu(p_{5})+e^{+}(p_{6}))\\ \lambda^{-}(\rightarrow\nu^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{3})+\bar{\nu}(p_{k}))\\ \lambda^{-}(\rightarrow\nu^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ \lambda^{-}(\rightarrow\nu^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu(p_{5})+\mu^{+}(p_{6}))\\ \lambda^{-}(\rightarrow\nu^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu^{-}(p_{3})+\mu^{+}(p_{4}))\\ \lambda^{-}(\rightarrow\nu^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu^{-}(p_{3})+\mu^{+}(p_{4}))\\ \lambda^{-}(\rightarrow\nu^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu^{-}(p_{3})+\mu^{+}(p_{4}))\\ \lambda^{-}(\rightarrow\nu^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu^{-}(p_{3})+\mu^{+}(p_{4}))\\ \lambda^{-}(\rightarrow\nu^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu^{-}(p_{3})+\mu^{+}(p_{4}))\\ \lambda^{-}(\rightarrow\nu^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu^{-}(p_{5})+\mu^{-}(p_{4}))\\ \lambda^{-}(\rightarrow\nu^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu^{-}(p_{5})+\mu^{-}(p_{4}))\\ \lambda^{-}(\rightarrow\nu^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu^{-}(p_{5})+\mu^{-}(p_{4}))\\ \lambda^{-}(\rightarrow\nu^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu^{-}(p_{5})+\mu^{-}(p_{4}))\\ \lambda^{-}(\rightarrow\nu^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\nu^{-}(p_{5})+\mu^{-}(p_{4}))\\ \lambda^{-}(\rightarrow\nu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5}))\\ \lambda^{-}(\rightarrow\nu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})\\ \lambda^{-}(\nu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})\\ \lambda^{-}(\nu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})\\ \lambda^{-}(\nu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})\\ \lambda^{-}(\nu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})\\ \lambda^{-}(\nu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})\\ \lambda^{-}(\nu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})\\ \lambda^{-}(\nu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{$	NLO NLO NLO NLO NLO NLO NLO NLO NLO NLO
72 W 73 W 76 W 77 W 81 Z' 82 Z' 83 Z' 84 Z' 86 Z' 87 Z' 88 Z' 89 Z' 90 W	$\begin{array}{l} V^{+}(\rightarrow\nu(p_{3})+\mu^{+}(p_{k}))+Z^{0}(\rightarrow\nu_{e}(p_{5})+\bar{\nu}_{e}(p_{5}))\\ V^{+}(\rightarrow\nu(p_{3})+\mu^{+}(p_{k}))+Z^{0}(\rightarrow b(p_{5})+\bar{b}(p_{5}))\\ V^{-}(\rightarrow\mu^{-}(p_{3})+\bar{\nu}(p_{k}))+Z^{0}(\rightarrow e^{-}(p_{5})+e^{+}(p_{6}))\\ V^{-}(\rightarrow\mu^{-}(p_{3})+\bar{\nu}(p_{k}))+Z^{0}(\rightarrow\nu(p_{5})+\bar{\nu}(p_{6}))\\ V^{-}(\rightarrow e^{-}(p_{3})+\bar{\nu}(p_{k}))+Z^{0}(\rightarrow b(p_{5})+\bar{b}(p_{6}))\\ V^{-}(\rightarrow e^{-}(p_{3})+\bar{\nu}(p_{k}))+Z^{0}(\rightarrow b(p_{5})+\bar{b}(p_{6}))\\ V^{-}(\rightarrow 3\times(\nu(p_{3})+\bar{\nu}(p_{k})))+Z^{0}(\rightarrow e^{-}(p_{5})+e^{+}(p_{6}))\\ V^{-}(\rightarrow 3\times(\nu(p_{3})+\bar{\nu}(p_{k})))+Z^{0}(\rightarrow b(p_{3})+\bar{b}(p_{k}))\\ V^{-}(\rightarrow 3\times(\nu(p_{3})+\bar{\nu}(p_{k})))+Z^{0}(\rightarrow b(p_{5})+\bar{b}(p_{6}))\\ V^{-}(\rightarrow e^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow\mu^{-}(p_{3})+\mu^{+}(p_{k}))\\ V^{-}(\rightarrow e^{-}(p_{5})+e^{+}(p_{6})+E^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})\\ V^{-}(\rightarrow e^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})\\ V^{-}(\rightarrow e^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})\\ V^{-}(\rightarrow e^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})\\ V^{-}(\rightarrow e^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})\\ V^{-}(\rightarrow e^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})\\ V^{-}(\rightarrow e^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})\\ V^{-}(\rightarrow e^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})+\mu^{-}(p_{5})\\ V^{-}(\rightarrow $	NLO NLO NLO NLO NLO NLO NLO NLO NLO
73 W 76 W 77 W 78 W 81 Z' 83 Z' 84 Z' 86 Z' 88 Z' 89 Z' 91 W 96 W	$\begin{array}{l} V^{+}(\to\nu(p_{3})+\mu^{+}(p_{4}))+Z^{0}(\to b(p_{5})+\bar{b}(p_{6}))\\ V^{-}(\to\mu^{-}(p_{3})+\bar{\nu}(p_{4}))+Z^{0}(\to e^{-}(p_{5})+e^{+}(p_{6}))\\ V^{-}(\to e^{-}(p_{3})+\bar{\nu}(p_{4}))+Z^{0}(\to\nu(p_{5})+\bar{\nu}(p_{6}))\\ V^{-}(\to e^{-}(p_{3})+\bar{\nu}(p_{4}))+Z^{0}(\to b(p_{5})+\bar{b}(p_{6}))\\ V^{-}(\to e^{-}(p_{3})+\mu^{+}(p_{4}))+Z^{0}(\to b(p_{5})+\bar{b}(p_{6}))\\ V^{-}(\to a^{-}(p_{3})+\mu^{+}(p_{4}))+Z^{0}(\to e^{-}(p_{5})+e^{+}(p_{6}))\\ V^{-}(\to e^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\to b(p_{3})+\bar{b}(p_{4}))\\ V^{-}(\to e^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\to b(p_{3})+\bar{b}(p_{4}))\\ V^{-}(\to a^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\to b(p_{5})+\mu^{+}(p_{6}))\\ V^{-}(\to e^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\to b(p_{5})+\mu^{+}(p_{4}))\\ V^{-}(\to e^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\to b(p_{5})+\mu^{+}(p_{4}))\\ V^{-}(\to e^{-}(p_{5})+e^{+}(p_{5}))+Z^{0}(\to b(p_{5})+\mu^{+}(p_{4}))\\ V^{-}(\to e^{-}(p_{5})+e^{+}(p_{5}))+Z^{0}(\to b(p_{5})+\mu^{+}(p_{4}))\\ V^{-}(\to e^{-}(p_{5})+e^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{4}))\\ V^{-}(\to e^{-}(p_{5})+e^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{4}))\\ V^{-}(\to e^{-}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{4}))\\ V^{-}(\to e^{-}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})\\ V^{-}(\to e^{-}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})\\ V^{-}(\to e^{-}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})\\ V^{-}(\to e^{-}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})\\ V^{-}(\to e^{-}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})\\ V^{-}(\to e^{-}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})\\ V^{-}(\to e^{-}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})\\ V^{-}(\to e^{-}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})\\ V^{-}(\to e^{-}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})\\ V^{-}(\to e^{-}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})\\ V^{-}(\to e^{-}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})\\ V^{-}(\to e^{-}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})\\ V^{-}(\to e^{-}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})+\mu^{+}(p_{5})\\ V^$	NLO NLO NLO NLO NLO NLO NLO NLO
76	$\begin{array}{l} Z^{-}(\rightarrow \mu^{-}(p_{3}) + \bar{\nu}(p_{4})) + Z^{0}(\rightarrow e^{-}(p_{5}) + e^{+}(p_{6})) \\ Z^{-}(\rightarrow e^{-}(p_{3}) + \bar{\nu}(p_{4})) + Z^{0}(\rightarrow \nu(p_{5}) + \bar{\nu}(p_{6})) \\ Z^{-}(\rightarrow e^{-}(p_{3}) + \bar{\nu}(p_{4})) + Z^{0}(\rightarrow b(p_{5}) + \bar{b}(p_{6})) \\ Z^{0}(\rightarrow \mu^{-}(p_{3}) + \mu^{+}(p_{4})) + Z^{0}(\rightarrow e^{-}(p_{5}) + e^{+}(p_{6})) \\ Z^{0}(\rightarrow a^{-}(p_{5}) + \mu^{+}(p_{4})) + Z^{0}(\rightarrow e^{-}(p_{5}) + e^{+}(p_{6})) \\ Z^{0}(\rightarrow e^{-}(p_{5}) + e^{+}(p_{6})) + Z^{0}(\rightarrow b(p_{3}) + \bar{b}(p_{4})) \\ Z^{0}(\rightarrow a^{-}(p_{5}) + e^{+}(p_{6})) + Z^{0}(\rightarrow \mu^{-}(p_{5}) + \mu^{+}(p_{6})) \\ Z^{0}(\rightarrow e^{-}(p_{5}) + e^{+}(p_{6})) + Z^{0}(\rightarrow \mu^{-}(p_{3}) + \mu^{+}(p_{4})) \\ Z^{0}(\rightarrow e^{-}(p_{5}) + e^{+}(p_{6})) + Z^{0}(\rightarrow \mu^{-}(p_{3}) + \mu^{+}(p_{4})) \\ Z^{0}(\rightarrow e^{-}(p_{5}) + e^{+}(p_{6})) + Z^{0}(\rightarrow \mu^{-}(p_{3}) + \mu^{+}(p_{4})) \end{array}$	NLO NLO NLO NLO NLO NLO NLO
77	$\begin{array}{l} V^-(\to e^-(p_3) + \bar{\nu}(p_4)) + Z^0(\to \nu(p_5) + \bar{\nu}(p_6)) \\ V^-(\to e^-(p_3) + \bar{\nu}(p_4)) + Z^0(\to b(p_5) + \bar{b}(p_6)) \\ v^0(\to \mu^-(p_3) + \mu^+(p_4)) + Z^0(\to e^-(p_5) + e^+(p_6)) \\ v^0(\to 3 \times (\nu(p_3) + \bar{\nu}(p_4))) + Z^0(\to e^-(p_5) + e^+(p_6)) \\ v^0(\to e^-(p_5) + e^+(p_6)) + Z^0(\to b(p_3) + \bar{b}(p_4)) \\ v^0(\to 3 \times (\nu(p_3) + \bar{\nu}(p_4))) + Z^0(\to b(p_5) + \bar{b}(p_6)) \\ v^0(\to e^-(p_5) + e^+(p_6)) + Z^0(\to \mu^-(p_3) + \mu^+(p_4)) \left[\ln \gamma^* \right] \end{array}$	NLO NLO NLO NLO NLO NLO
78	$\begin{array}{l} \swarrow (-) e^{-}(p_3) + \bar{\nu}(p_4)) + Z^0 (\rightarrow b(p_5) + \bar{b}(p_6)) \\ 0 (\rightarrow \mu^{-}(p_3) + \mu^{+}(p_4)) + Z^0 (\rightarrow e^{-}(p_5) + e^{+}(p_6)) \\ 0 (\rightarrow 3 \times (\nu(p_3) + \bar{\nu}(p_4))) + Z^0 (\rightarrow e^{-}(p_5) + e^{+}(p_6)) \\ 0 (\rightarrow e^{-}(p_5) + e^{+}(p_6)) + Z^0 (\rightarrow b(p_3) + \bar{b}(p_4)) \\ 0 (\rightarrow e^{-}(p_5) + e^{+}(p_6)) + Z^0 (\rightarrow b(p_3) + \bar{b}(p_6)) \\ 0 (\rightarrow e^{-}(p_5) + e^{+}(p_6)) + Z^0 (\rightarrow \mu^{-}(p_3) + \mu^{+}(p_4)) [\text{no } \gamma^*] \end{array}$	NLO NLO NLO NLO NLO
81 Z' 82 Z' 83 Z' 84 Z' 86 Z' 87 Z' 88 Z' 91 W 96 W	$ \begin{array}{l} ^{0}(\rightarrow \mu^{-}(p_{3}) + \mu^{+}(p_{4})) + Z^{0}(\rightarrow e^{-}(p_{5}) + e^{+}(p_{6})) \\ ^{0}(\rightarrow 3 \times (\nu(p_{3}) + \bar{\nu}(p_{4}))) + Z^{0}(\rightarrow e^{-}(p_{5}) + e^{+}(p_{6})) \\ ^{0}(\rightarrow e^{-}(p_{5}) + e^{+}(p_{6})) + Z^{0}(\rightarrow b(p_{3}) + \bar{b}(p_{4})) \\ ^{0}(\rightarrow 3 \times (\nu(p_{3}) + \bar{\nu}(p_{4}))) + Z^{0}(\rightarrow b(p_{5}) + \bar{b}(p_{6})) \\ ^{0}(\rightarrow e^{-}(p_{5}) + e^{+}(p_{6})) + Z^{0}(\rightarrow \mu^{-}(p_{3}) + \mu^{+}(p_{4})) \left[\text{no } \gamma^{*} \right] \\ \end{array} $	NLO NLO NLO NLO
82 Z 83 Z 84 Z 86 Z 87 Z 88 Z 89 Z 91 W 96 W	$ \begin{array}{l} ^{0}(\rightarrow 3\times (\nu(p_{3})+\nu(p_{4})))+Z^{0}(\rightarrow e^{-}(p_{5})+e^{+}(p_{6})) \\ o(\rightarrow e^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow b(p_{3})+\bar{b}(p_{4})) \\ o(\rightarrow 3\times (\nu(p_{3})+\bar{\nu}(p_{4})))+Z^{0}(\rightarrow b(p_{5})+\bar{b}(p_{6})) \\ o(\rightarrow e^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow \mu(p_{3})+\mu^{+}(p_{4})) \left[\text{no } \gamma^{*} \right] \\ \end{array} $	NLO NLO NLO
83 Z ⁰ 84 Z ⁰ 86 Z ⁰ 87 Z ⁰ 88 Z ⁰ 89 Z ⁰ 91 W 96 W	$ \begin{array}{l} {}^{0}(\rightarrow e^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow b(p_{3})+\bar{b}(p_{4})) \\ {}^{0}(\rightarrow 3\times (\nu(p_{3})+\bar{\nu}(p_{4})))+Z^{0}(\rightarrow b(p_{5})+\bar{b}(p_{6})) \\ {}^{0}(\rightarrow e^{-}(p_{5})+e^{+}(p_{6}))+Z^{0}(\rightarrow \mu^{-}(p_{3})+\mu^{+}(p_{4})) \ [\text{no } \gamma^{*}] \end{array} $	NLO NLO
84 Z ⁰ 86 Z ⁰ 87 Z ⁰ 88 Z ⁰ 89 Z ⁰ 91 W 96 W	$ \begin{array}{l} {}^{0}(\rightarrow 3 \times (\nu(p_{3}) + \bar{\nu}(p_{4}))) + Z^{0}(\rightarrow b(p_{5}) + \bar{b}(p_{6})) \\ {}^{0}(\rightarrow e^{-}(p_{5}) + e^{+}(p_{6})) + Z^{0}(\rightarrow \mu^{-}(p_{3}) + \mu^{+}(p_{4})) \text{ [no } \gamma^{*}] \end{array} $	NLO
86 Z ⁰ 87 Z ⁰ 88 Z ⁰ 89 Z ⁰ 91 W 96 W	$0 \rightarrow e^{-}(p_5) + e^{+}(p_6) + Z^0 \rightarrow \mu^{-}(p_3) + \mu^{+}(p_4) \pmod{\gamma^*}$	_
87	$C^0(\to e^-(p_5) + e^+(p_6)) + Z^0(\to \mu^-(p_3) + \mu^+(p_4)) [\text{no } \gamma^*]$	NLO
87	$0(\rightarrow e^{-(n_r)} + e^{+(n_r)}) + Z^0(\rightarrow 3 \times (y(n_r) + \bar{y}(n_r)))$ [no α^*]	
89 Z ⁰ 91 W 96 W	$(\neg e (p_5) + e (p_6)) + Z (\neg g \land (\nu(p_3) + \nu(p_4))) \text{III} $	NLO
91 W 96 W	$0 \rightarrow e^{-}(p_5) + e^{+}(p_6) + Z^0 \rightarrow b(p_3) + \overline{b}(p_4) $ [no γ^*]	NLO
91 W 96 W	$^{0}(\rightarrow 3 \times (\nu(p_{3}) + \bar{\nu}(p_{4}))) + Z^{0}(\rightarrow b(p_{5}) + \bar{b}(p_{6})) \text{ [no } \gamma^{*}]$	NLO
	$V^{+}(\to \nu(p_3) + e^{+}(p_4)) + H(\to b(p_5) + b(p_6))$	NLO
	$V^-(\rightarrow e^-(p_3) + \bar{\nu}(p_4)) + H(\rightarrow b(p_5) + \bar{b}(p_6))$	NLO
	$0 \rightarrow e^{-(p_3)} + e^{+(p_4)} + H \rightarrow b(p_5) + b(p_6)$	NLO
$102 Z^0$	$0 \rightarrow 3 \times (\nu(p_3) + \bar{\nu}(p_4)) + H(\rightarrow b(p_5) + \bar{b}(p_6))$	NLO
	$0 \rightarrow b(p_3) + \overline{b}(p_4) + H \rightarrow b(p_5) + \overline{b}(p_6)$	NLO
	$I(\rightarrow b(p_3) + \bar{b}(p_4))$	NLO
	$I(\to \tau^-(p_3) + \tau^+(p_4))$	NLO
	$H(\to W^+(\nu(p_3) + e^+(p_4)) + W^-(e^-(p_5) + \bar{\nu}(p_6)))$	NLO
	$I(\to Z^0(\mu^-(p_3) + \mu^+(p_4)) + Z^0(e^-(p_5) + e^+(p_6))$	NLO
	$I(\to Z^0(3 \times (\nu(p_3) + \bar{\nu}(p_4))) + Z^0(e^-(p_5) + e^+(p_6))$	NLO
		NLO
	$(1 \rightarrow Z^*)(I_1 \cup I_2) + (I_1 \cup I_2) + Z^*(0)(I_2) + 0(I_2)$	
	$I(\to Z^0(\mu^-(p_3) + \mu^+(p_4)) + Z^0(b(p_5) + \bar{b}(p_6))$ $I(\to b(p_3) + \bar{b}(p_4)) + b(p_5)(+g(p_6))$	I NLO
143 H	$\begin{array}{c} (1 \rightarrow Z - (\mu - (p_3) + \mu^+ (p_4)) + Z - (b(p_5) + b(p_6)) \\ \overline{((\rightarrow b(p_3) + \overline{b}(p_4)) + b(p_5)(+g(p_6))} \\ \overline{((\rightarrow b(p_3) + \overline{b}(p_4)) + \overline{b}(p_5)(+b(p_6))} \end{array}$	NLO NLO

MCFM Information

Version 4.0 available at:

```
http://mcfm.fnal.gov
```

- Improvements over previous releases:
 - ⋆ more processes
 - * better user control
 - ★ separate variation of factorization and renormalization scales
 - ★ support for PDFLIB, Les Houches PDF accord
 (→ PDF uncertainties)
 - ⋆ ntuples as well as histograms
 - unweighted events
 - ⋆ basic event generator interface
 - ⋆ 'Behind-the-scenes' efficiency

suitable for LO calculations

Not to be confused with ...

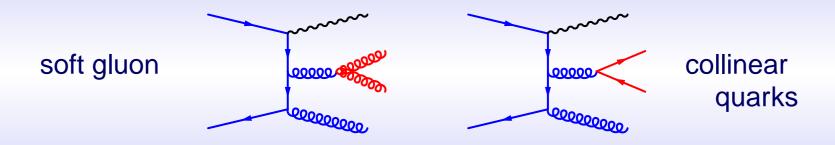
... other results you might find on Google:

```
More **** For the Masses - MCFM www.mc-fm.com
```

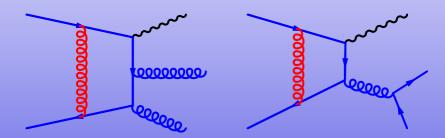
"Go on folks - say it loud, say it proud! MCFM is here to carry you into the next dimension where you'll be exposed to an affair of pure grunt, an explosion of energy, an uncut declaration of life as it is!"

Vector boson + jets in MCFM

- Many diagrams, sensitive to all parton PDF's
- NLO corrections are separated into two classes:
- REAL extra parton radiation, e.g.



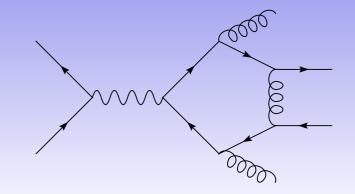
■ VIRTUAL loop diagrams:



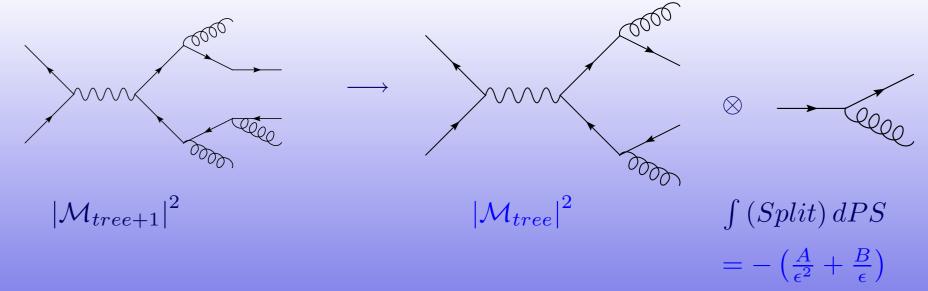
NLO basics

VIRTUAL

$$\int d^{4-2\epsilon} \ell \ 2\mathcal{M}_{loop}^* \mathcal{M}_{tree}$$
$$= \left(\frac{A}{\epsilon^2} + \frac{B}{\epsilon}\right) \left| \mathcal{M}_{tree} \right|^2$$



REAL



Subtraction method

- Methods for dealing with singular regions are well-developed, such as phase-space slicing and dipole subtraction
- MCFM uses the dipole subtraction method, which makes a careful choice of the kinematics in the subtraction terms to improve the cancellation of singularities

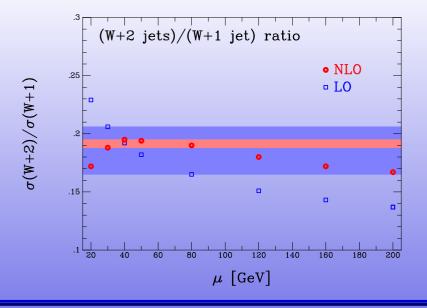
Catani, Seymour

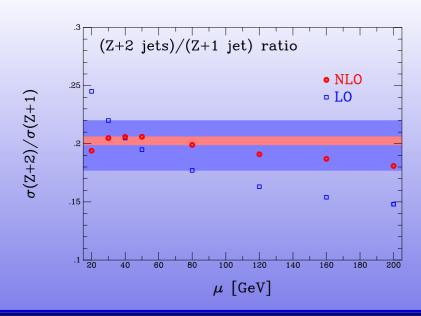
- However, for high multiplicity final states, the number of singular regions is large, resulting in:
 - ★ Very many dipoles, each with their own set of kinematics
 - ★ Time-consuming calculation of subtraction terms
- Modifications to the original formalism have been implemented in MCFM. The subtraction region can be limited in order to speed up the code and also to improve numerical stability

Z. Nagy, hep-ph/0307268

W/Z+ jet cross-sections

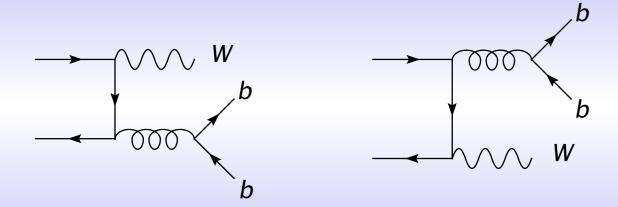
- The W/Z+2 jet NLO calculation is the most complicated (time-consuming) process currently implemented. This is due to both the lengthy virtual matrix elements (vector boson +4 partons) and the complicated structure of phase space.
- The usual features such as reduced scale dependence are observed, e.g. the theoretical prediction for the number of events containing 2 jets divided by the number with only 1 is improved.





Vector boson + heavy flavour in MCFM

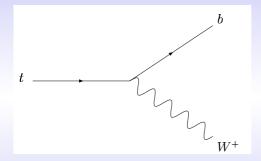
In lowest order b-quark pairs are produced in association with W's by gluon splitting alone:



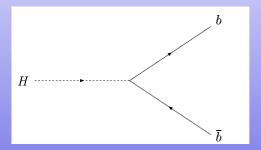
- Beyond LO, the *b*-quark is treated as a massless particle in MCFM
 - \star a finite cross-section requires a cut on the b-quark p_T
 - ★ this means that this calculation is not suitable for estimating the rate with only a single b tag

Heavy flavour as a background

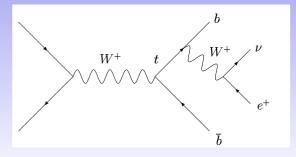
- Events containing jets that are heavy-quark tagged are important for understanding both old and new physics:
 - ★ Top decays $t \rightarrow W + b$

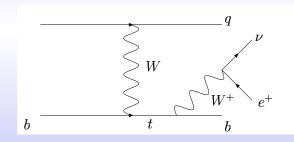


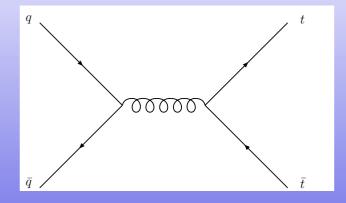
 \star Much new physics couples preferentially to massive quarks, for instance a light Higgs with $m_H < 140$ GeV decaying to $b\bar{b}$



Top processes







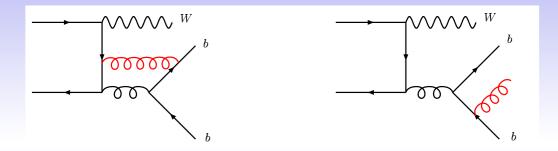


 \longrightarrow 2 jets, only one is a b

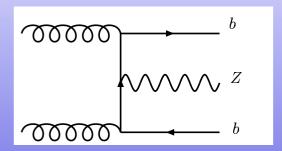
 \longrightarrow ≥ 2 jets, two are b's

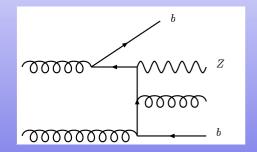
Heavy flavour beyond lowest order

At NLO, the simple kinematics can be altered:



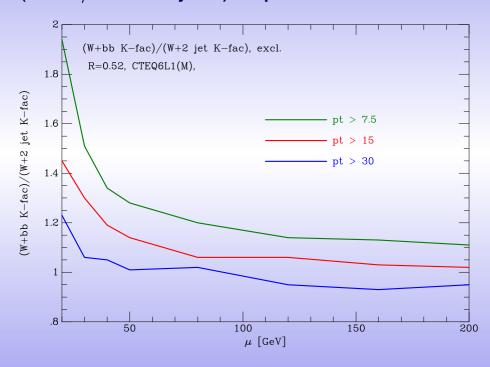
■ For heavy flavour production in association with a Z, the b-quarks do not have be produced by gluon splitting. Beyond LO, the difference is further magnified.





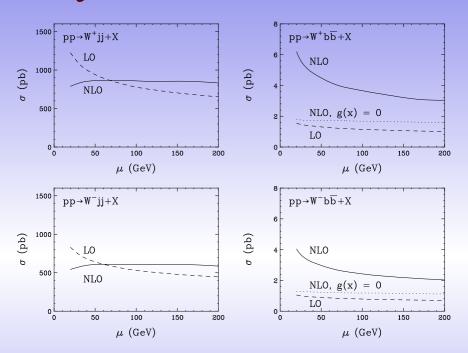
K-factor ratio

Important for CDF's "Method 2". Essentially, is a lowest order estimate of $(Wb\bar{b}/W + 2 \text{ jets})$ reproduced at NLO?

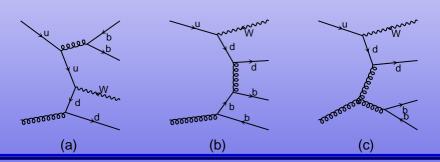


- A qualified "yes" it is for scale choices around 50 GeV or greater and p_T cuts of about 15 GeV or greater.
- This is true for these cuts, at the Tevatron

Jets and heavy flavour at the LHC



■ The large gluonic contribution appearing in $Wb\bar{b}$ for the first time at NLO results in a huge correction and poor scale dependence.

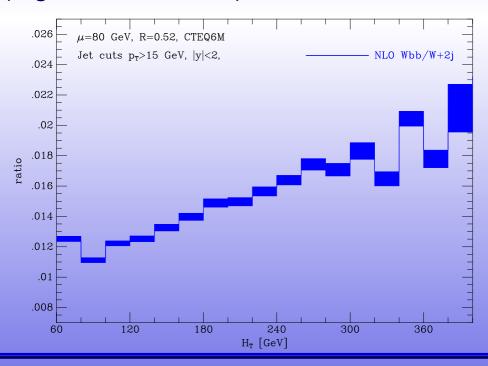


PDF uncertainties

Total cross-section uncertainty:

$$Wbar{b}
ightarrow 2.5\%$$
 , $W+2j
ightarrow 1.5\%$.

■ Implemented using the LHAPDF interface, with the phase space integration driven by the central PDF set and weights for the remainder (e.g. 40 for CTEQ6) accumulated concurrently

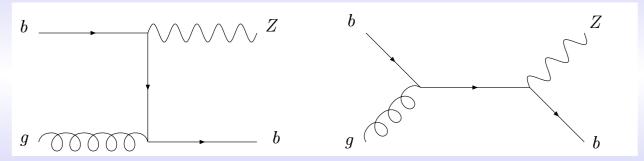




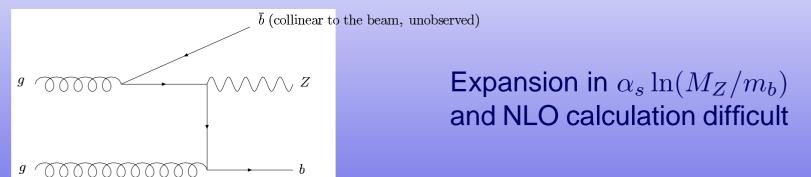
JC, Ellis, Maltoni, Willenbrock

Heavy flavour fraction revisited

- Often the presence of two b-quarks in the final state is actually only inferred from a single b-tag
- In this case, there is another way of computing the theoretical cross-section. For instance, in the case of Z+ heavy flavour:



■ Requires knowledge of *b*-quark pdf's, but compare to:



Z + b at NLO - Run II

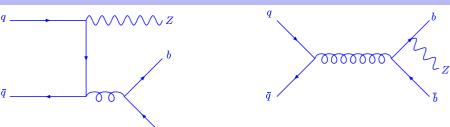
JC, K. Ellis, F. Maltoni and S. Willenbrock, hep-ph/0312024

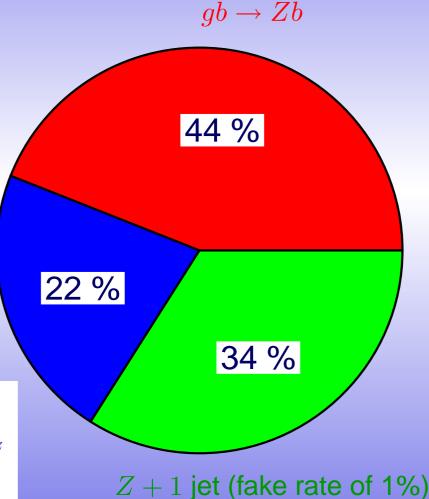


$$\sigma(Z + \text{ one } b \text{ tag}) = 20 \text{ pb}$$

- Fakes from Z+ jet events are significant
- Prediction for ratio of Z + b to untagged $Z + \mathrm{jet}$ is 0.02 ± 0.004

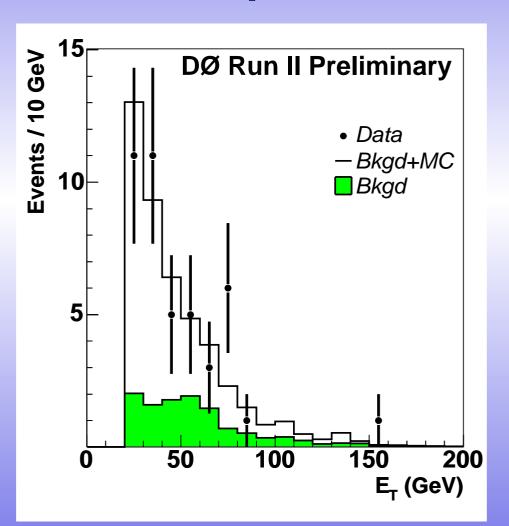






Experimental result

■ Based on 189 pb⁻¹ of data from Run II



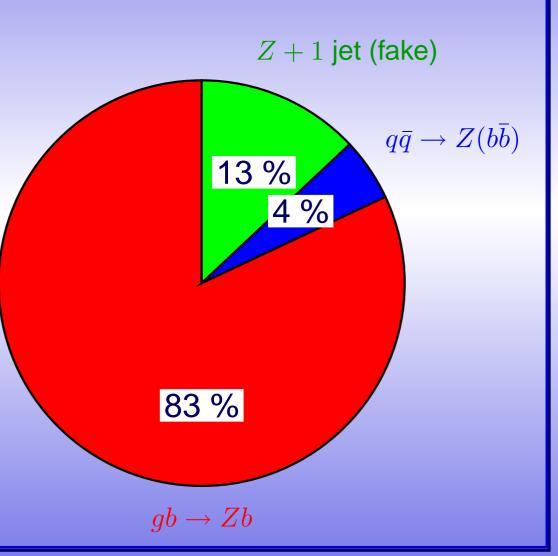
Preliminary ratio of crosssections:

$$\frac{\sigma(Z+b)}{\sigma(Z+j)} = 0.024 \pm 0.007$$

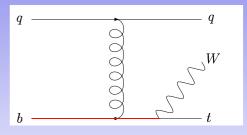
compatible with the NLO prediction from MCFM

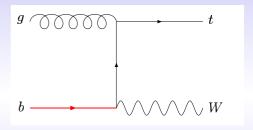
LHC expectations

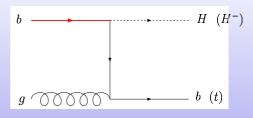
- $ho_T^{
 m jet} > 15$ GeV, $|\eta^{
 m jet}| < 2.5$
- $\sigma(Z + \text{ one } b \text{ tag}) = 1 \text{ nb}$
- Fakes from Z+ jet events are much less significant and $q\bar{q}$ contribution is tiny
- This should allow a fairly clean measurement of heavy quark PDF's (currently, only derived perturbatively)

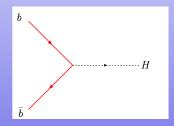


b-PDF uses







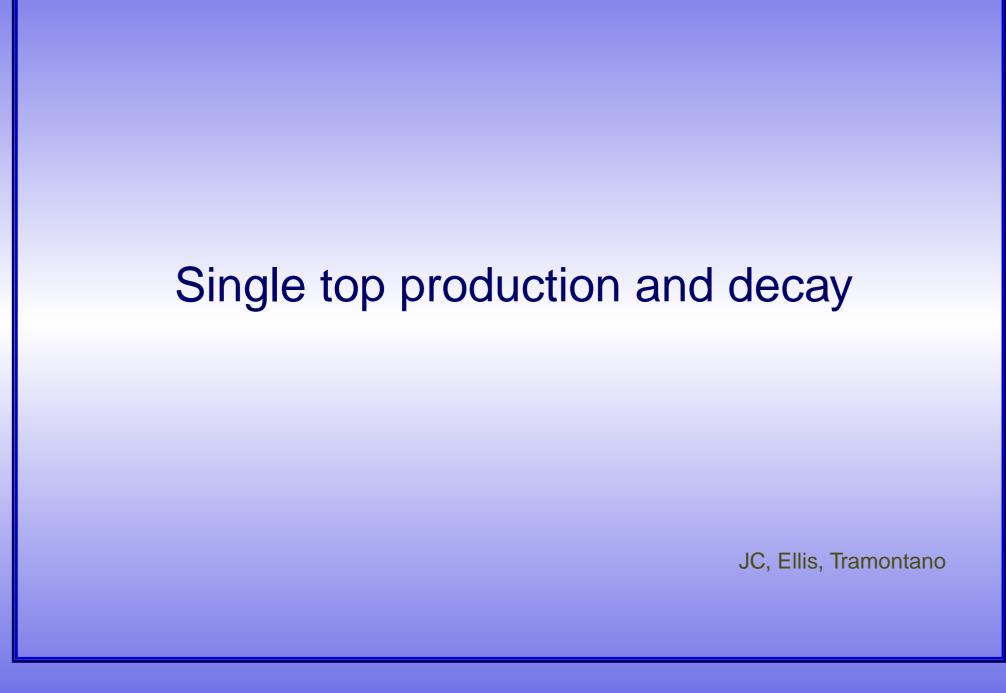


single-top $qb \rightarrow qtW$

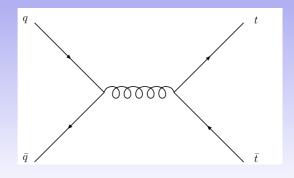
single-top $gb \rightarrow tW$

(charged) Higgs+b

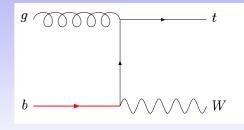
inclusive Higgs



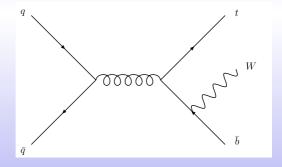
Top production processes



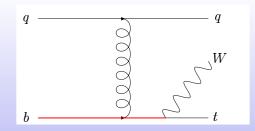
6 pb 720 pb



0.08 pb 50 pb



0.8 pb 10 pb

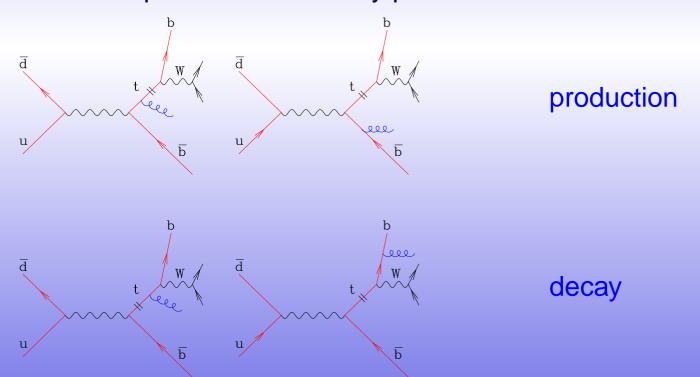


1.8 pb 240 pb

- All cross-sections known to NLO (Tevatron / LHC)
- Single top cross-section smaller by a factor of 2, but yields further information about the top quark due to the weak interaction involved, e.g. a direct determination of V_{tb}

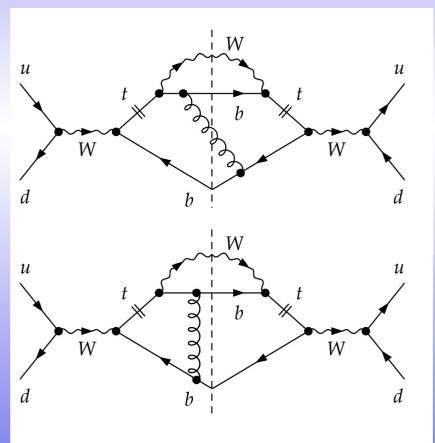
Inclusion of decay

- Results had previously been presented without including the decay of the top quark. Without it, predictions for some quantities used in Tevatron search strategies are impossible
- Final state radiation that enters at next-to-leading order is possible in either the production or decay phase:



On-shell approximation

- Such a division is only possible assuming that every diagram has one top quark on its mass-shell, neglecting $\mathcal{O}(\Gamma_t/m_t)$ effects
- Contributions not included are interferences of the form:



real

virtual

Justification

Fadin, Khoze, Martin Melnikov, Yakovlev

- Characteristic time scale for production of the top quark is of order $1/m_t$, whereas the time for the decay is $1/\Gamma_t$
- In general, these stages are well-separated and interference effects average to zero
- If the gluon is soft, this argument is not sufficient
- However, in this region real and virtual radiation must cancel so that this phase space is not especially enhanced. In order for the propagator to stay resonant, the gluon energy must be of $\mathcal{O}(\Gamma_t)$
- For IR-safe variables, interference effects should be $\mathcal{O}(\alpha_s\Gamma_t/m_t)$
- Quantitatively confirmed for the s-channel process and for $t\bar{t}$ production in e^+e^- annihilation Pittau; Macesanu

Implementation in MCFM

- Uses the extension of the dipole method to handle massive particles
 Catani, Dittmaier, Seymour, Trocsanyi
- Extra subtraction term to deal with radiation in the decay of the top quark
 - ★ Real process containing soft and collinear singularities

$$t \longrightarrow W + b + g$$

★ Counter-term

$$t \longrightarrow \widetilde{W} + \widetilde{b}$$

 \star Since the W is implemented with a Breit-Wigner, we use a Lorentz transformation which ensures that $\tilde{p}_W^2 = p_W^2$ and also ensure that the b-quark remains massless $\tilde{p}_b^2 = p_b^2 = 0$ JC, Ellis, Tramontano

Results

Parton level study of the Tevatron single top analysis performed

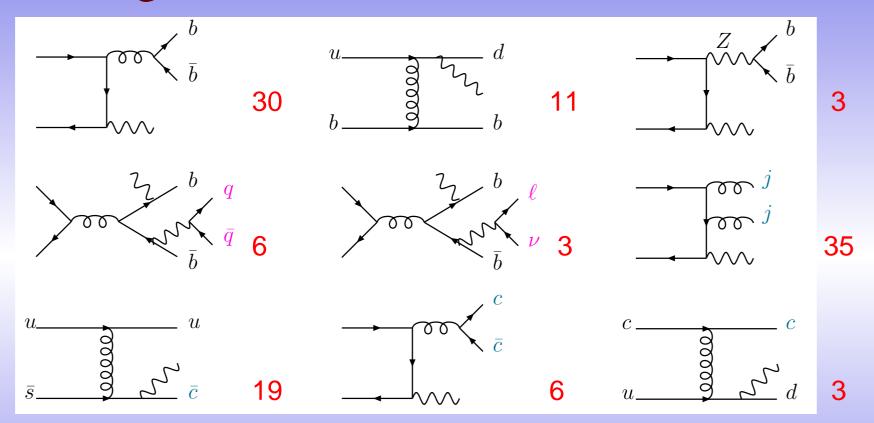
by CDF

Lepton p_T	$p_T^e > 20 \; \mathrm{GeV}$
Lepton pseudorapidity	$ \eta^e < 1.1$
Missing E_T	
$Jet\ p_T$	$p_T^{ m jet} > 15~{ m GeV}$
Jet pseudorapidity	$ \eta^{jet} < 2.8$
Mass of $b + l + \nu$	$140 < m_{bl\nu} < 210 \text{ GeV}$

■ The inclusion of radiation in the decay lowers the (exclusive two-jet) cross-section slightly:

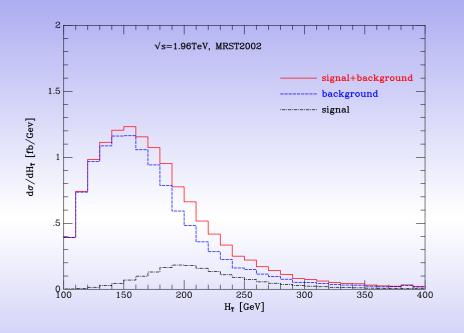
Process	σ_{LO} [fb]	σ_{NLO} [fb]
s-channel single top	10.3	11.7
s-channel (with decay radiation)	10.3	11.3
t-channel single top	38.8	29.4
t-channel (with decay radiation)	38.8	26.6

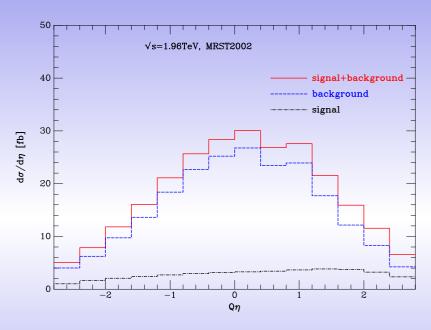
Backgrounds



- Cross-sections in fb include nominal tagging efficiences and mis-tagging/fake rates. Calculated with MCFM, most at NLO
- Rates are 7 fb and 11 fb for s- and t-channel signal

Single top signal vs. backgrounds





- $\blacksquare H_T = \text{scalar sum of jet, lepton and missing } E_T$
- $\blacksquare Q\eta$ is the product of the lepton charge and the rapidity of the untagged jet, useful for picking out the t-channel process
- Signal:Background (with our nominal efficiencies) is about 1:6
 - a very challenging measurement indeed

Shortcomings

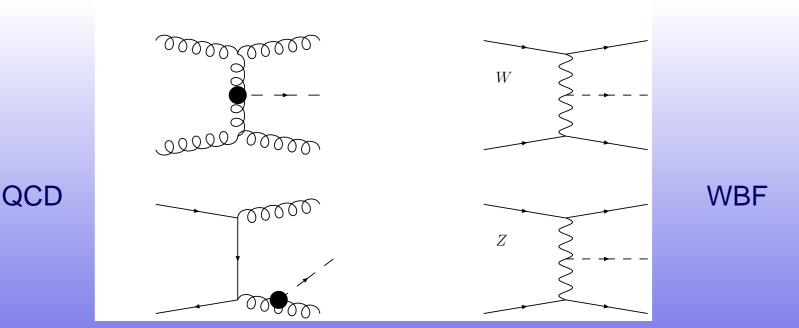
The approach in MCFM involves a number of approximations:

- The b-quark is massless LO calculation with $m_b = 4.75~{\rm GeV} \longrightarrow <1\%$ effect
- The top quark is put on its mass-shell LO calculation with a Breit-Wigner $\longrightarrow 1\%$ effect
- We neglect interference between radiation in production/decay qualitative argument for $\mathcal{O}(\alpha_s\Gamma_t/m_t)\sim$ less than a percent
- We assume p_T -independent heavy flavour tagging efficiencies, as well as stable b and c quarks easily addressed by a more detailed experimental analysis now that the code is publicly available
- No showering or hadronization is performed no NLO/PS prediction yet available; however the large cone size $\Delta R=1$ should help minimize these effects



WBF introduction

- Scenario: a SM-like Higgs boson has been discovered with a mass $115 < m_H < 200~{\rm GeV}$
- At the LHC, a sample of events containing a Higgs boson and two jets has been isolated
- This sample contains Higgs bosons produced both by pure QCD processes and by weak boson fusion (WBF)



Terminology

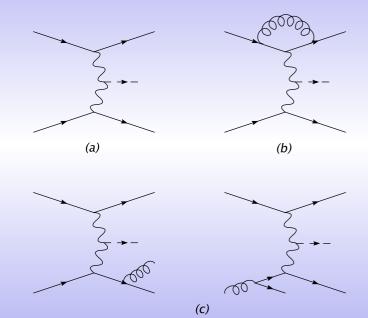
- One can take advantage of the production mode in WBF to measure the coupling of the Higgs boson to the vector bosons W and Z
- In that context, one can regard the QCD production of H+2 jets as an irreducible background that must be determined as accurately as possible for a good determination of the couplings
- How well can this measurement be done and how well are these processes modelled?
- Other backgrounds that, for instance, do not involve a Higgs boson, must surely be considered also. However, an absolute bound on the accuracy is given by the ability to differentiate between WBF and QCD production

NLO calculation

- The calculation is quite straightforward, involving only vertex corrections due to no colour exchange between the lines
- K-factor is only 1.1, with small effects in most distributions
- Results are in good agreement with both previous inclusive and differential results

Figy, Oleari, Zeppenfeld

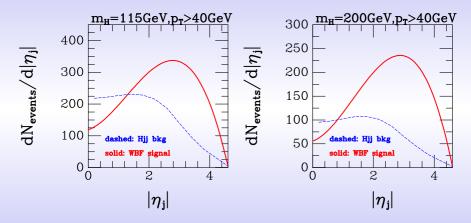
■ The residual scale uncertainty is about 2%, whilst that due to (CTEQ) PDF's is about 3%



■ The evidence indicates that the signal is calculated reliably, with an error of the order of 5%

Event characteristics

■ WBF events are expected to produce a central Higgs boson with the two jets at large values of p_T and rapidity. In contrast, QCD H+2 jets production generates jets with a softer p_T spectrum and which are also more central



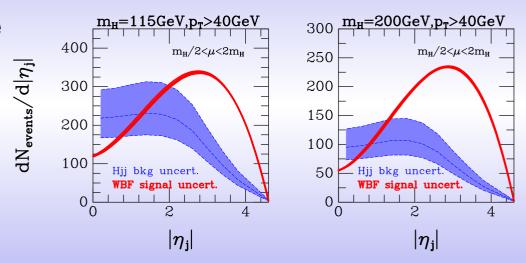
Berger, JC

- A variety of cuts have been proposed to exploit this difference. We simply insist that one jet lies in the rapidity range $1.6 < \eta < 4.4$
- Although the QCD process produces soft jets, the WBF region typically contains jets of only moderate p_T . Therefore an accurate representation of the QCD contribution requires hard matrix elements and jets produced from a shower are not sufficient

Uncertainties

The background H+2 jets process is only known at leading order. The matrix elements implemented in MCFM include an effective ggH coupling in the limit $m_t\gg m_H$ Kauffman, Desai, Risal (c.f. Del Duca, Kilgore, Oleari, Schmidt, Zeppenfeld)

- This results in very large scale dependence $(\alpha_s^4 \text{ process})$
- Integrating over the WBF region gives a background uncertainty of +70% and -40%



- The background is substantial. NLO K-factors for more inclusive processes are large (about 1.8 for $pp \to H$ and 1.4 for $pp \to H + 1$ jet) and can only make the situation worse
- The uncertainty in the background manifests itself directly in the expected uncertainty on the coupling determination

Sample purity

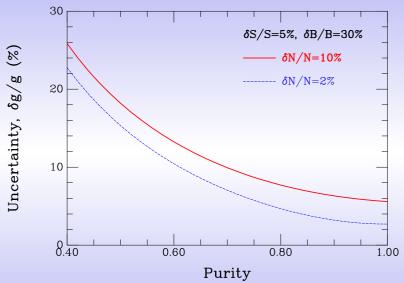
■ Event rates and sample purity, defined by P = S/(S + B), can change greatly depending on the jet p_T cut [rates in fb]

p_T cut [GeV]	20	40	80
Signal ($m_H = 115$)	1374	789	166
Bkg	1196	382	92
Purity	0.53	0.67	0.64
Signal ($m_H = 200$)	928	545	121
Bkg	534	179	46
Purity	0.63	0.75	0.72

- A p_T cut of 40 GeV gives quite a good purity over the whole Higgs mass range. A lower cut roughly doubles the rate, but at a substantial loss of purity
- We can evaluate the uncertainty on the coupling measurement $(\delta g/g)$ in terms of the signal and background uncertainties $(\delta S/S, \delta B/B)$, the statistical uncertainty $(\delta N/N)$ and the purity P

Expected precision

- $lacktriangleright \delta N/N$ is based on the rates in the WBF region, after appropriate decays are accounted for. We examine two scenarios:
 - \star $\delta N/N \sim 0.1$ roughly appropriate for $10 {\rm fb}^{-1}$ of integrated luminosity
 - * $\delta N/N \sim 0.02 \text{ for } 100 \text{fb}^{-1}$
 - \star $\delta S/S$ from before $\delta B/B$ fairly optimistic



- Purities of ~ 0.7 yield uncertainties in the 5-10% range
- If the NLO K-factor were ~ 1.6 then $P \sim 0.55$; this would cause the uncertainty to deteriorate to 10-15%
- Determination is limited by background uncertainty, not statistics

Summary

■ The program MCFM provides NLO predictions for many processes of interest at the Tevatron and the LHC

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http://mcfm.fnal.gov/
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- W+jets: well-behaved at NLO, comparison with data is being studied at the Tevatron
- W+heavy flavour: $Wb\bar{b}$ suffers from large QCD corrections at the LHC due to new gluon-initiated processes
- **Z**+heavy flavour: recent progress at D0 in measuring the fraction of Z + b in Z+ jets; looks promising for measuring the b-quark PDF at the LHC
- Single top: inclusion of full spin correlations and gluon radiation in the top quark decay allows a better comparison with experimental analyses; the multitude of backgrounds looks very daunting
- WBF: extraction of Higgs-vector boson couplings looks difficult without a NLO calculation of the H+2 jet QCD process